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ECONOMICS

Paper : ECOHC2046

( **Mathematical Methods for Economics—I** )

*Full Marks : 80*

*Pass Marks : 32*

*Time : 3 hours*

*The figures in the margin indicate full marks  
for the questions*

1. Choose the correct answer of the following  
(any six) : 1×6=6
- (a) If  $U = \{5, 6, 7, 8, 9\}$  and  $A = \{5, 6\}$ , then  
complement of set A, i.e., ( $A^c$ ) is
- (i)  $\{5, 6, 7, 8\}$
  - (ii)  $\{5, 6\}$
  - (iii)  $\{7, 8, 9\}$
  - (iv)  $\{ \}$
- (b) Under the identity law of set theory  
 $A \cap \phi =$
- (i)  $A^c$
  - (ii)  $\phi$
  - (iii)  $A$
  - (iv)  $0$

- (c)  $\int \frac{1}{2} dx$  gives
- (i)  $\frac{1}{2} + C$
  - (ii)  $\frac{1}{2}x + C$
  - (iii) 0
  - (iv) None of the above
- (d) If  $y = f(x) = b$ , then  $\lim_{x \rightarrow M} b$  is equal to
- (i)  $b$
  - (ii)  $M$
  - (iii)  $\infty$
  - (iv) 0
- (e) What is the Cartesian product of  $A \times B$  if  $A = \{x, y\}$  and  $B = \{1, 2\}$ ?
- (i)  $\{(x, 1), (y, 2)\}$
  - (ii)  $\{(x, 1), (x, 2), (y, 1), (y, 2)\}$
  - (iii)  $\{(1, x), (2, x), (1, y), (2, y)\}$
  - (iv)  $\{(y, 1), (y, 2), (x, 1), (x, 2)\}$
- (f) When the slope of average cost (AC) is zero, i.e.,  $\frac{d}{dQ}(AC) = 0$ , then
- (i)  $MC < AC$
  - (ii)  $MC > AC$
  - (iii)  $MC = AC$
  - (iv) None of the above

- (g) If  $y = f(x) = 10^x$ , then  $\frac{dy}{dx}$  gives
- (i)  $\frac{1}{10^x}$
  - (ii)  $10^x \log_e 10$
  - (iii)  $x \log_e 10$
  - (iv) 0
- (h) The necessary and sufficient conditions for minimization of the function  $y = f(x)$  is
- (i)  $\frac{dy}{dx} = 0$  and  $\frac{d^2x}{dy^2} > 0$
  - (ii)  $\frac{dy}{dx} = 0$  and  $\frac{d^2x}{dy^2} < 0$
  - (iii)  $\frac{dy}{dx} = 0$  and  $\frac{d^2x}{dy^2} = 0$
  - (iv) None of the above
- (i)  $\int a^x dx$  gives
- (i)  $\frac{a}{\log_e a} + C$
  - (ii)  $\frac{a^x}{\log_e a} + C$
  - (iii)  $\frac{a}{a \log_e a} + C$
  - (iv) None of the above

(j) Which one of the following is an irrational number?

(i)  $\frac{4}{5}$

(ii) 2.3123457...

(iii) 1.404040...

(iv) 1.14

2. Answer any five of the following questions :

$$2 \times 5 = 10$$

(a) Evaluate :

$$\lim_{x \rightarrow 1} \frac{x^2 + 7x - 8}{(x-1)}$$

(b) Evaluate  $\int_{-1}^3 (2x^2 + 5) dx$ .

(c) Define exponential function.

(d) If  $A = \{1, 2, 3, 4\}$  and  $B = \{3, 4, 5, 6\}$ , represent  $A \cap B$  in the Venn diagram.

(e) Define point of inflection.

(f) Find the second-order derivative of the function

$$f(x) = 15x^3 - 6x^2 + 30x + 20$$

(g) The average revenue (AR) function is given by  $AR = 100 - 3Q$ , find the elasticity of demand when  $Q = 5$ .

3. Answer any six of the following questions :

$$5 \times 6 = 30$$

(a) Find  $\frac{dy}{dx}$  of the following functions :  $2+3=5$

(i)  $y = f(x) = \frac{A}{\sqrt{x}}$

(ii)  $y = f(x) = (7x^2 + 2x - 20)^7$

(b) Given the consumption function

$$C = 500 - \frac{2000}{5 + Y}$$

where  $Y$  is income, find marginal propensity to consume (MPC) when  $Y = 45$ . Also find marginal propensity of save (MPS). 3+2=5

(c) Find the equilibrium price ( $\bar{P}$ ) and equilibrium quantity ( $\bar{Q}$ ) from the given simple market model :

$$Q_d = a - bP$$

$$Q_s = -c + dP$$

$$Q_d = Q_s$$

(d) The total cost function of a firm is given by

$$TC = 200Q - 5Q^2 + 0.05Q^3$$

where  $Q$  is the quantity produced. Find out the output at which marginal cost (MC) is equal to the average cost (AC).

- (e) Find the extreme value of the function  
 $y = 2x^2 - 24x + 10$
- (f) Examine the continuity of the function  
 $f(x) = x^2 - 2x + 3$ , when  $x < 1$   
 $= 1$ , when  $x = 1$   
 $= 2x^2 - 3x + 5$ , when  $x < 1$
- (g) Integrate the following : 2+3=5  
 (i)  $\int (2ax + 10\sqrt{x} + e^x + 50) dx$   
 (ii)  $\int \frac{2x+3}{x^2+3x} dx$
- (h) Define the concepts of quasi-convex and quasi-concave.
- (i) A firm's production is given by  
 $Q = f(L) = -\frac{2}{3}L^3 + 10L^2$   
 where  $L$  is the labour employed. Show that diminishing marginal product of labour operates when employment of labour is 6.
- (j) The total cost function of a firm is given by  
 $C(Q) = 0.005Q^3 - 0.02Q^2 + 30Q + 3000$   
 Find—  
 (i) average cost (AC) when  $Q = 10$ ;  
 (ii) marginal cost (MC) when  $Q = 10$ .

2+3=5

4. Answer any *two* of the following questions : 10×2=20
- (a) Derive the mathematical relationship between average revenue, marginal revenue and price elasticity of demand using differentiation.
- (b) (i) Given the demand function,  $Q = a - bP$ , find the total revenue function.  
 (ii) If the consumer's demand function is given by  
 $Q = f(P) = \sqrt{60 - 2P}$   
 find the consumer's surplus when market price  $P = 12$ . 3+7=10
- (c) State and prove the quotient rule of differentiation. 3+7=10
- (d) For the market model  
 $D = a - bP$   
 $S = -c + d(P - t)$   
 $D = S$   
 where  $D$ ,  $S$ ,  $P$  and  $t$  denote demand, supply, price and rate of sale tax or excise duty. Find the optimal tax rate which maximizes tax revenue.

5. Answer any one of the following questions : 14

(a) In a Cobweb model

$$Q_{dt} = a - bP_t \quad (a, b > 0)$$

$$Q_{st} = -c + dP_{t-1} \quad (c, d > 0)$$

$$Q_{dt} = Q_{st}$$

Obtain the time path of price  $P$  and also analyze the three situations arising depending upon the slopes of demand and supply curves.

8+6=14

(b) A monopolist has the following total revenue (TR) and total cost (TC) functions :

$$TR = R(Q) = 30Q - Q^2$$

$$TC = C(Q) = Q^3 - 15Q^2 + 10Q + 100$$

Find—

(i) profit maximizing output;

(ii) equilibrium price;

(iii) point elasticity of demand at equilibrium level of output. 10+2+2=14

(c) (i) The marginal revenue and total cost functions of a monopolist are

$$MR = 20 - 2Q \text{ and } TC = Q^2 + 8Q + 2$$

Find the profit maximizing output.

(ii) Given the Cobb-Douglas production function  $Q = AL^\alpha K^{1-\alpha}$ , find the marginal physical productivity of labour and capital respectively.

6+4+4=14

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