63/1 (SEM–2) ECO HC 2046 (CC 4)

2022

ECONOMICS

Paper : CC–4

(Mathematical Methods for Economics—I)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×6=6

(a) If $A = \{a, b, c, d\}$ and $B = \{c, d, e, f\}$, then $A \cap B$ is

(i) $\{a, b, c, d\}$

(ii) $\{c, d\}$

(iii) $\{c, d, e, f\}$

(iv) None of the above

(b) $y = f(x) = b^x$ is a/an

(i) constant function

(ii) polynomial function

(iii) exponential function

(iv) logarithmic function

(c) If the total revenue function is given as $R = 2x^2 - 10x$, then marginal revenue (MR) is

(i) $x^2 - 5$

(ii) $4x$

(iii) $4x - 10$

(iv) $2x^3 - 10x^2$

(d) The derivative of $y = e^{3x}$ is

(i) e^{3x}

(ii) $3e^{3x}$

(iii) xe^{3x}

(iv) None of the above

(e) The differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} = xy$$

is of order

(i) 2

(ii) 1

(iii) 4

(iv) 0

(f) Integration of $\int e^{ax}\, dx$ gives

(i) $e^{ax} + C$

(ii) $ae^{ax} + C$

(iii) $\frac{1}{a}e^{ax} + C$

(iv) None of the above

2. Answer the following questions : 2×5=10

(a) Why do we add one constant term while integrating a function?

(b) The total cost of producing x units of a commodity is given by

$$C(x) = 1000 + 300x + x^2$$

find the total cost of producing 100 units of a commodity, i.e., $C(100)$.

(c) Define constant function. Give one economic example of constant function. 1+1=2

(d) Define domain of a function.

(e) If two sets are $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 4, 5, 6\}$, then find $A - B$.

3. Answer any *six* of the following questions : 5×6=30

(a) Solve the differential equation

$$\frac{dy}{dx} + 5y = 8$$

given $y(0) = 3$.

(b) "Derivative or differentiation is a mathematical tool used to find out the magnitude and direction of change in a particular variable due to change in the value of other variable." Explain the statement with some economic examples.

(c) Find $\frac{dy}{dx}$ of the following functions : 2+3=5

(i) $y = \frac{x^{1000}}{1000}$

(ii) $y = \frac{x+1}{x-1}$

(d) Explain the local and global maxima and minima with the help of a diagram of the function, $y = f(x)$.

(e) Find the equilibrium price and quantity of the following market model :

$$Q_d = 20 - 2 \cdot 5p$$
$$Q_s = -4 + 1 \cdot 5p$$
$$Q_d = Q_s$$

(f) From the given total cost function, $TC = 2Q^2 + 5Q + 18$, find the output at which average cost is minimum.

(g) Define continuity of a function. State why a function must be continuous for it to be conformable of differentiation.

(h) State the necessary and sufficient conditions for maximization and minimization of the function, $y = f(x)$.

(i) Explain relative extrema and absolute extrema.

4. Answer any *two* of the following questions : 10×2=20

(a) Integrate the following : 2+4+4=10

(i) $\int (a^2 + 2ax - x^2)\,dx$

(ii) $\int \frac{(2x+2)}{(x^2 + 2x - 10)^3}\,dx$

(iii) $\int \frac{1}{(7x-5)^2}\,dx$

(b) Given the total cost function

$$C = Q^3 - 5Q^2 + 14Q + 75$$

write out a total variable cost (TVC) function. Find the derivative of the total variable cost function (TVC) and interpret the economic meaning of that derivative. 2+5+3=10

(c) The total revenue (R) and total cost (C) of a firm are functions of output (q) such that $R = R(q)$ and $C = C(q)$. Explain the conditions for obtaining optimum level of output of a firm under perfect competition market using the necessary and sufficient conditions of maximization.

5. Answer any *one* of the following questions : 14

(a) A firm has the following total cost and demand functions

$$C = \frac{1}{3}Q^3 - 7Q^2 + 111Q + 50$$

$$Q = 100 - P$$

(i) Write out the total revenue function (R).

(ii) Formulate the total profit function (π).

(iii) Find the profit maximizing level of output $\overline{Q}$.

(iv) What is the maximum profit? 2+2+8+2=14

(b) Show the relationship between average cost (AC) and marginal cost (MC) using the product and quotient rules of differentiation. 7+7=14

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