63/1 (SEM–4) COM HG 4046 (GE 4)

2022

COMMERCE

Paper : GE–4

(Business Mathematics)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following : 1×6=6

(a) Unit matrix is a

(i) diagonal matrix

(ii) scalar matrix

(iii) Both *(i)* and *(ii)*

(iv) None of the above

(b) If x denotes the volume of output, then $\frac{d}{dx}(TR)$ is equal to

(i) MR

(ii) AR

(iii) 0

(iv) None of the above

(c) If any two rows (or columns) in a determinant are identical, then the value of determinant will

(i) remain same

(ii) change by sign

(iii) be zero

(iv) None of the above

(d) $\int \frac{1}{x} dx$ is equal to

(i) $a^x + c$

(ii) $e^x + c$

(iii) $0 + c$

(iv) $\log x + c$

(e) If interest is compounded half-yearly, then the formula for finding amount is

(i) $A = P\left(1 + \frac{r}{100}\right)^n$

(ii) $A = P\left(1 + \frac{r}{2 \times 100}\right)^{2n}$

(iii) $A = P\left(1 + \frac{r}{3 \times 100}\right)^{3n}$

(iv) $A = P\left(1 + \frac{r}{4 \times 100}\right)^{4n}$

(f) The graphical method is used to solve linear programming problem when the number of decision variables is

(i) 6

(ii) 2

(iii) 4

(iv) 5

2. Answer the following questions : 2×5=10

(a) Write two differences between a matrix and a determinant.

(b) If $y = x^3 - \frac{1}{x^2}$, then find $\frac{dy}{dx}$.

(c) Integrate

$$\int \frac{\log x}{x} dx$$

(d) If

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 3 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 5 & 6 \\ 1 & 1 & -2 \end{bmatrix}$$

then find $2A - 3B$.

(c) If any two rows (or columns) in a determinant are identical, then the value of determinant will

(i) remain same

(ii) change by sign

(iii) be zero

(iv) None of the above

(d) $\int \frac{1}{x} dx$ is equal to

(i) $a^x + c$

(ii) $e^x + c$

(iii) $0 + c$

(iv) $\log x + c$

(e) If interest is compounded half-yearly, then the formula for finding amount is

(i) $A = P\left(1 + \frac{r}{100}\right)^n$

(ii) $A = P\left(1 + \frac{r}{2 \times 100}\right)^{2n}$

(iii) $A = P\left(1 + \frac{r}{3 \times 100}\right)^{3n}$

(iv) $A = P\left(1 + \frac{r}{4 \times 100}\right)^{4n}$

(f) The graphical method is used to solve linear programming problem when the number of decision variables is

(i) 6

(ii) 2

(iii) 4

(iv) 5

2. Answer the following questions : 2×5=10

(a) Write two differences between a matrix and a determinant.

(b) If $y = x^3 - \frac{1}{x^2}$, then find $\frac{dy}{dx}$.

(c) Integrate

$$\int \frac{\log x}{x} dx$$

(d) If

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 3 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 5 & 6 \\ 1 & 1 & -2 \end{bmatrix}$$

then find $2A - 3B$.

(e) Find the interest on ₹730 from 15th March to 1st September (of the same year) @ 5% per annum.

3. Answer any *six* of the following questions : 5×6=30

(a) Solve the following system of simultaneous linear equations by Cramer's rule :

$$2x+3y-z=15$$
$$4y+2z=16$$
$$3x+2y=18$$

(b) Without expanding, prove that

$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

(c) If $f(x)=x^2-3x+2$, then find $f(A)$, where A is a matrix given by

$$\begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 4 \\ -1 & 1 & 0 \end{bmatrix}$$

(d) (i) The total cost function $C(x)$ of producing x items is given by

$$C(x)=\begin{cases} 1000+5x; & \text{when } 0 \le x \le 500 \\ 2000+4x; & \text{when } 500 < x \le 2000 \end{cases}$$

Find the cost of producing (1) 430 items and (2) 1200 items. 2

(ii) If

$$f(x)=\frac{ax+b}{bx+a}$$

then prove that $f(x)\cdot f\left(\frac{1}{x}\right)=1$. 3

(e) Evaluate :

(i) $\lim\limits_{x\to 2}\dfrac{x^2-x-2}{x^2-4}$

(ii) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$

(f) If

$$y=2x+\frac{4}{x}$$

then prove that

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$$

(g) Verify Euler's theorem for the function $f(x, y) = x^3 + 2x^2y + y^3$.

(h) Define objective function, feasible region and feasible solution associated with linear programming.

(i) A machine depreciated in value each year 10% of its previous value and at the end of fourth year its value was ₹ 1,31,220. Find the original value of the machine.

4. Answer any *two* of the following questions : 10×2=20

(a) Solve the following system of equations by using matrix inversion method :

$$3x + 4y + 5z = 18$$
$$2x - y + 8z = 13$$
$$5x - 2y + 7z = 20$$

(b) For a monopoly firm producing a certain article can sell x articles per week at p rupees each, where $5x = 375 - 3p$. The cost of production is $\left(500 + 13x + \frac{1}{5}x^2\right)$ rupees. Find how many articles the monopolist should produce for maximum profit. What is that maximum profit?

(c) (i) If $u = 2(ax + by)^2 - (x^2 + y^2)$, then show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 4(a^2 + b^2) - 4$$ 5

(ii) The marginal cost of producing x units of a product is given by $(25 + 30x - 9x^2)$ and the fixed cost is known to be 55. Find the total cost and average cost functions. 5

5. Answer any *one* of the following questions : 14

(a) (i) What is linear programming problem? What are the basic assumptions of linear programming problem? 2+5=7

(ii) Solve the following LPP by graphical method : 7

Maximize $Z = 3x + 4y$

subject to

$$4x + 2y \leq 80$$
$$2x + 5y \leq 180$$
$$x \geq 0, y \geq 0$$

(b) (i) What is sinking fund? A sinking fund is created for the redemption of debentures of ₹ 1,00,000 at the end of 25 years. How much money should be provided out of profits each year for the sinking fund, if the investment can earn interest @ 4% per annum?

[Given, $\log 1 \cdot 04 = 0 \cdot 0170$, antilog $0 \cdot 425 = 2 \cdot 661$] 2+6=8

(ii) At what rate percent will ₹ 6,000 amount to ₹ 7,500 in 4 years? [Given, $\log 1 \cdot 25 = 0 \cdot 0969$, antilog $0 \cdot 0242 = 1 \cdot 057$] 6

★★★