*Total number of printed pages–12*

**63/1 (Sem–4) GE4/COMHG4046**

**2024**

**COMMERCE**

Paper : COMHG4046

***(Business Mathematics)***

*Full Marks : 80*

*Pass Marks : 32*

Time : Three hours

***The figures in the margin indicate full marks for the questions.***

1. Choose the most appropriate option for ***any six*** of the following questions : 1×6=6

   *(a)* Identity matrix is a :

   *(i)* square matrix

   *(ii)* diagonal matrix

   *(iii)* scalar matrix

   *(iv)* All of the above

*Contd.*

(b) If $A^T = A$ then the matrix $A$ is said to be :

(i) skew-symmetric matrix

(ii) symmetric matrix

(iii) triangular matrix

(iv) singular matrix

(c) If $u$ and $v$ are two differentiable functions of $x$ then :

(i) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} + u\frac{dv}{dx}}{v^2}$

(ii) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

(iii) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u\frac{dv}{dx} - v\frac{du}{dx}}{v^2}$

(iv) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{u^2}$



(d) $\int (M.R.)dx$ is equal to

(i) M. R.

(ii) A. R.

(iii) T. R.

(iv) All of the above

(e) If the interest is payable quarterly then the formula for amount $(A)$ at compound interest (C. I.) is

(i) $A = P(1 + \frac{r}{100})^n$

(ii) $A = P(1 + \frac{r}{2\times100})^{2n}$

(iii) $A = P(1 + \frac{r}{3\times100})^{3n}$

(iv) $A = P(1 + \frac{r}{4\times100})^{4n}$

Where, $P$ = Principal; $r$ = Rate of interest percent p.a; $n$ = Time in year.

(f) In an LPP, the restrictions or limitations under which the objective function is to be optimized is called

(i) Constraints

(ii) Objective function

(iii) Decision variables

(iv) Feasible solutions

(g) Le $A$ and $B$ be two matrices then the product $AB$ exist if

(i) No. of rows in $A$ = No. of rows in $B$

(ii) No. of rows in $A$ = No. of columns in $B$

(iii) No. of columns in $A$ = No. of rows in $B$

(iv) No. of columns in $A$ = No. of columns in $B$

(h) Graphical method is used to solve linear programming problems when the number of decision variables is

(i) 6

(ii) 2

(iii) 4

(iv) 5

(i) $\frac{d}{dx}(\sqrt{x})$ is equal to

(i) $2\sqrt{x}$

(ii) $\frac{\sqrt{x}}{2}$

(iii) $\frac{1}{2\sqrt{x}}$

(iv) $\frac{2}{\sqrt{x}}$



(j) In a triangular matrix

(i) No. of rows = No. of columns

(ii) No. of rows < No. of columns

(iii) No. of rows > No. of columns

(iv) All of the above

2. Answer the following questions : 2×5=10

(a) Define rectangular matrix and square matrix.

(b) The simple interest on a sum is equal to $\frac{1}{10}$ of itself in 4 years. Find the rate of interest.

(c) Write *two* differences between a matrix and a determinant.

(d) Define limit of a function.

(e) The total cost function $C(x)$ of producing $x$ items is given by

$$C(x) = \begin{cases} 1000 + 5x; & \text{when } 0 \le x \le 500 \\ 2000 + 4x; & \text{when } 500 < x \le 2000 \end{cases}$$

Find the cost of producing *(i)* 430 items *(ii)* 1200 items.

(f) Define the following associated with linear programming problem :

Objective function, Decision variables

(g) Find the derivative of $8x^3 - 3x^2 + 12x - 5$ with respect to $x$.

3. Answer ***any six*** of the following questions : 5×6=30

(a) Solve the following equation:

$$\begin{vmatrix} a & a & a \\ b & x & x \\ c & b & a \end{vmatrix} = 0 ; \text{ where } a \neq b.$$

(b) If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}$ then find (i) $AA^T$ (ii) $A^TA$ and hence show that $AA^T \neq A^TA$.

(c) (i) If $f(x) = \frac{3x^4 + 5x^2 + 3}{x^2}$ then prove that $f\left(\frac{1}{x}\right) = f(x)$. 2

(ii) Evaluate :

$$\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 + x - 6}$$ 3



(d) If $xy = ae^x - be^{-x}$, prove that

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy = 0$$

(e) What is Economic Order Quantity (EOQ) ? State the assumptions of the EOQ model with uniform demand. 2+3=5

(f) Find all the first order and second order partial derivatives of

$u = 8x^2 + 6xy + 2y^2$

and verify that

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$$

(g) Find the following integrals : 2+3=5

(i) $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$

(ii) $\int \left(6e^x - 2x^3 + \frac{5}{x^2}\right) dx$

(h) Find the compound interest on Rs. 16,000 at 5% per annum at the end of $1\frac{1}{2}$ years if the interest is calculated half-yearly.

(i) A machine costing Rs 50,000 depreciates at a certain rate of 8% per annum. What is the depreciation charge for the 8th year ? If the estimated useful life of the machine is 10 years, determine its scrape value.

[Given, $(0.92)^7 = 0.5578466$; $(0.92)^8 = 0.5132188$ and $(0.92)^{10} = 0.4343884$]

(j) What are the requirement for a linear programming problem ?

4. Answer ***any two*** of the following questions : 10×2=20

(a) (i) Solve the following system of simultaneous linear equations by Cramer's rule : 6

$2x - 3y + z = 7;\ 2x + y - z = 1;$

$4y + 3z = -11$

(ii) If $A = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$ then find $2A + 3B$ and $2A - 3B$. 4



(b) A monopolist has a demand curve $x = 106 - 2p$ and average cost curve is $AC = 5 + \frac{x}{50}$ where $p$ is the price per unit output and $x$ is the number of units of output. If the total revenue is $R = xp$, determine the most profitable output and maximum profit.

(c) (i) Verify Euler's theorem for the following functions : 5

$f(x,y) = x^2 + y^2 + 3xy$

(ii) Given that $M.C. = 2 + 60q - 5q^2$ where $q$ denotes the units of commodity produced. Find the total cost function and average cost function if fixed cost is 65. 5

(d) (i) Mr. X borrows a certain sum of money at 8% p.a. compound interest and agrees to pay both the principal and the interest in 10 equal yearly installments of Rs 1200 each. If the first installment is to be paid at the end of 5 years from the date of borrowing and the other yearly installments are paid regularly at the end of the subsequent years, find the sum borrowed by him. 5

[Given, *log*(1.08)=0.0334; *Antilog* (0.3340)=2.158 and *Antilog* (0.4676)=2.935]

(ii) A machine costs a company Rs. 80,000 and its effective life is estimated to be 20 years. A sinking fund is created for replacing the machine at the end of its effective lifetime when its scrap realizes a sum of Rs. 5,000 only. Calculate to the nearest hundreds of rupees, the amount which should be provided every year, for the sinking fund if it accumulates at 9% p.a. compounded annually.

[Given, $log(1.09) = 0.0374$; $Antilog(0.7480) = 5.598$] 5

5. Answer ***any one*** of the following questions : 14×1=14

(a) (i) What are the basic assumptions of linear programming? Mention the limitations of linear programming problem. 5+3=8



(ii) Solve graphically the following LPP : 6

Maximize $Z = 10x + 15y$

subject to : $2x + y \leq 26$

$2x + 4y \leq 56$

$x - y \geq -5$

$x \geq 0,\ y \geq 0$

(b) (i) Show that the maximum value of $x + \frac{1}{x}$ is less than its minimum value. 5

(ii) A monopolist firm has the following marginal revenue function *(MR)* and total cost function *(C(x))* :

$MR = 20 - 2x$ and

$C(x) = x^2 + 8x + 2$

where $x$ is the number of unit produced. Find $x$ so that the profit is maximum. 9

(c) (i) An amount of Rs. 5,000 is put into three investments at the rates of interest of 6%, 7% and 8% per annum respectively. The total annual income is Rs. 358. If the combined income from the first two investments is Rs. 70 more than the income from the third, find the amount of each investment by using matrix algebra. 9

(ii) Three firms *A*, *B* and *C* supplied 40, 35 and 25 truck loads stones and 10, 5 and 8 truck loads of sand respectively to a contractor. If the costs of stones and sand are Rs. 1,200 and Rs. 500 per truck load respectively, find by using matrix method, the total amount paid by the contractor to each of these firms 5

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